# Effect of Drag and Frictional Losses on the Hydrodynamics of Gas-Lift Reactors

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In previous works (Sáez et al., 1998; Márquez et al., 1999a,b), a family of mathematical models was developed to describe the hydrodynamics of external loop, gas-lift reactors (GLRs) such as that shown in Figure 1. These models predict the rate of liquid circulation, and the axial profiles of gas holdup, pressure, bubble radius, composition, and gas and liquid velocities in the riser, in the bubbly flow regime. The Márquez et al. models (1999a,b) include the effects of chemical reaction and gas/liquid mass transfer, which can cause significant changes in gas holdup and bubble diameter along the length of the riser.

Two related approaches have been used in recent models of gas-lift reactors. The Sáez et al. (1998) and Márquez et al. (1999a,b) models followed the approach originally developed by Young et al. (1991); they are based on spatially averaged, one-dimensional equations of continuity and momentum in the riser. Other authors (such as García-Calvo and Letón, 1996; García-Calvo, 1992; Verlaan et al., 1986) have employed variations of the drift-flux model of Zuber and Findlay (1965) to model the riser. In both approaches, macroscopic mechanical-energy balances were written for the gas/liquid separator and external downcomer, making it necessary to use various correlations to estimate the drag and frictional losses in the system.

The purpose of this communication is to examine several assumptions concerning the drag and frictional losses in gaslift reactors that are common to the Sáez et al. (1998) and Márquez et al. (1999a,b) models, and to some other published models. These assumptions are as follows.

1. The friction factor in the downcomer must be adjusted upward with respect to correlations for fully developed, single-phase flow in pipes in order to account for developing flow. Even though their experiments showed no gas holdup in the downcomer, Young et al. (1991) found it necessary to increase the Moody friction factor by a multiple of 4 to 6,

depending on downcomer diameter. Sáez et al. (1998) and Márquez et al. (1999a,b) used Young et al.'s adjusted friction factors in their models. In contrast, friction factors for fully developed flow were used in other studies (García-Calvo and Letón, 1996; García-Calvo, 1992; Verlaan et al., 1986).

2. Mechanical energy losses in the gas/liquid separator and the straightening vanes can be neglected. None of the models

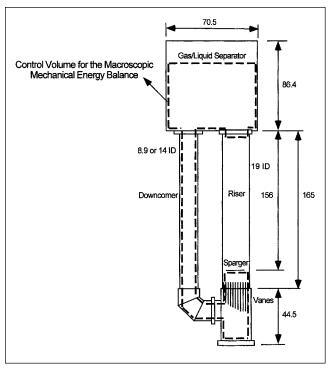


Figure 1. External-loop gas-lift reactor used by Young et al. (1991).

Separator chamber is 27.9 cm wide. Dimensions in cm.

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just referenced account for losses in straightening vanes. However, losses in the gas/liquid separator are considered in several models (García-Calvo and Letón, 1996; García-Calvo, 1992; Verlaan et al., 1986).

3. Frictional losses in the riser are adequately described by the correlation of Akita et al. (1988), which explicitly accounts for two-phase flow. Sáez et al. (1998) and Márquez et al. (1999a,b) used this correlation in their models. Friction factors for one-phase flow were used in the models of García-Calvo and Letón (1996), García-Calvo (1992), and Verlaan et al. (1986).

The reexamination of these assumptions was prompted by an anomaly that was noticed when the models of Sáez et al. (1998) and García-Calvo (1992) were used to represent experimental data taken by Young et al. (1991) in GLRs with two different downcomer diameters. Even though both models described the data reasonably well, they exhibited a significant difference in the quality of the predictions for the two downcomers. This suggests that the models, as formulated, do not adequately represent the effects of either geometry or liquid circulation velocity on mechanical energy losses in the system.

In this work we present an analysis of the impact that drag and frictional losses have on model predictions, and show how the appropriate quantification of these losses leads to accurate predictions of Young et al.'s data, regardless of downcomer size.

#### Model Formulation

The model presented in this section is similar to that developed in a previous work (Sáez et al., 1998). Details will be provided only when the present analysis differs from the previous formulation. The GLR to be modeled is the one used in the experiments reported by Young et al. (1991) and Márquez et al. (1999a) (Figure 1).

#### Riser continuity equations

In general, the gas holdup and gas liquid velocities will vary with axial position in the riser. At any point, the following continuity equations are satisfied: Liquid:

$$\rho_1(1-\bar{\epsilon})\langle \bar{v}_1 \rangle = L_f \tag{1}$$

Gas:

$$\frac{\langle \overline{P} \rangle M_g}{RT} \overline{\epsilon} \langle \overline{v}_g \rangle = G, \qquad (2)$$

where  $\epsilon$  is the gas holdup;  $v_1$  and  $v_g$  are liquid and gas velocities, respectively; P is pressure;  $\rho_1$  is liquid density;  $M_g$  is gas molecular weight; and  $L_f$  and G are liquid and gas mass flow rates per unit cross-sectional area. We have assumed no mass transfer between gas and liquid, so that  $L_f$  and G are uniform in the riser. Dispersive contributions are not considered, so that our results are constrained to bubbly flow. The angular brackets in Eqs. 1 and 2 represent intrinsic-phase volume averages, and the overbars represent averages over the cross section of the riser. Gas bubbles are assumed to be

spherical and monodisperse, with a bubble radius at the sparger (z = 0) of 1 mm.

#### Riser momentum equations

The momentum equations in the riser are (Sáez et al., 1998) as follows: Liquid:

$$\frac{d\langle \bar{P} \rangle}{dz} + (1 - \bar{\epsilon}) \rho_1 g + \frac{f_r}{D} \frac{1}{2} \rho_1 \langle \bar{v}_1 \rangle^2 = 0$$
 (3)

Gas:

$$\bar{\epsilon}g = \frac{1}{8}\bar{a}_{1g}\hat{C}_d(\langle \bar{v}_g \rangle - \langle \bar{v}_1 \rangle)|\langle \bar{v}_g \rangle - \langle \bar{v}_1 \rangle|, \tag{4}$$

where g is the acceleration of gravity;  $\bar{a}_{1g}$  is the gas/liquid interfacial area per unit volume;  $f_r$  is the two-phase wall friction factor; and  $\hat{C}_d$  is the drag coefficient for a bubble rising in the liquid.

## Mechanical-energy balance

The hydrodynamics of the rest of the system are represented by a mechanical-energy balance in a control volume that encompasses the gas/liquid separator, the downcomer, the connection between the downcomer and the riser, and the section of the riser below the sparger (Figure 1). In previous works (Young et al., 1991; Sáez et al., 1998; Márquez et al., 1999a,b), frictional losses associated with the gas/liquid separator were neglected.

The quantification of drag and frictional losses is crucial in the calculation of liquid circulation rates. All sources of mechanical-energy losses must be identified and quantified. For the GLR employed by Young et al. (1991), the sources of mechanical-energy losses, following the direction of liquid circulation from the top of the riser to the sparger, are as follows:

- 1. Expansion between the riser and the gas/liquid separator. This loss will be assumed to correspond to an expansion into an infinitely large tank, and is characterized by a loss coefficient  $K_{SE} = 1$ .
- 2. Contraction from the separator into the downcomer. We will consider the contraction between an infinitely large tank and the downcomer. The corresponding loss coefficient is  $K_{SC} = 0.5$ .
- 3. Frictional losses in the downcomer. These losses will be quantified through the friction factor for fully developed flow in a smooth pipe,  $f_{dc}$ , as obtained from Moody's correlation.
- 4. Losses in the elbow. The loss coefficient for a long radius 90° bend is  $K_E\!=\!0.35$ .
- 5. Losses due to the expansion of the flow from the downcomer into the riser (the downcomer is assumed to have a smaller diameter than the riser). These losses will be estimated by the loss coefficient for a sudden expansion:

$$K_{ET} = \left[1 - \left(\frac{D_{dc}}{D}\right)^2\right]^2,$$

where D and  $D_{dc}$  are the riser and downcomer diameters, respectively.

- 6. Since the connection between the downcomer and riser involves not only a change in diameter but also a change in direction, we will consider an additional loss due to a "tee" with 90° flow. The corresponding loss coefficient is  $K_T = 1.5$ .
- 7. The straightening vanes give rise to frictional losses that will be treated as those corresponding to fully developed flow in parallel ducts. Frictional losses are quantified by taking the distribution of the flow into the channels delimited by the vanes into account. Each channel has a friction factor calculated from Moody's correlation for smooth pipes, using the hydraulic diameter of the channel as the characteristic length. These calculations showed that frictional losses associated with the vanes, for the range of flow rates considered in our calculations, could be approximated by an equivalent loss coefficient of  $K_V=2.1$ .

The final form of the mechanical energy balance for the control volume shown in Figure 1 is

$$\left\{ f_{dc} \frac{L_{dc}}{D_{dc}} + K_{SC} + K_{E} + K_{ET} + \left(\frac{D_{dc}}{D}\right)^{4} \right. \\
\times \left[ \frac{K_{T}}{\left(1 - \bar{\epsilon}_{z=0}\right)^{2}} + \frac{K_{SE}}{\left(1 - \bar{\epsilon}_{z=L}\right)^{2}} + K_{V} \right] \right\} \frac{\bar{v}_{dc}^{2}}{2 g} \\
= \left[ \frac{1}{\left(1 - \bar{\epsilon}_{z=L}\right)^{2}} - \frac{1}{\left(1 - \bar{\epsilon}_{z=0}\right)^{2}} \right] \frac{\bar{v}_{dc}^{2}}{2 g} \\
- \frac{\langle \bar{P} \rangle_{z=0} - \langle \bar{P} \rangle_{z=L}}{\rho_{1} g} + L, \quad (5)$$

where all terms containing velocities have been expressed in terms of the downcomer velocity by using the continuity equation. The lefthand side of Eq. 5 is the sum of all frictional and drag losses in the control volume. The first term on the righthand side represents the kinetic energy change between the top and bottom of the riser. The second term is proportional to the pressure drop through the riser, and the last term represents gravitational contributions.

#### Analysis of the gas/liquid separator

The solution of the equations just presented requires a knowledge of the pressure at some point in the system. Sáez et al. (1998) assumed that the pressure at the top of the riser was equal to the pressure at the top of the downcomer, and that both pressures were equal to the hydrostatic pressure produced by the liquid height in the separator. A more rigorous analysis can be performed by applying a mechanical-energy balance to the gas/liquid separator. Considering the drag and frictional losses due to expansion and contraction, neglecting any frictional losses due to flow through the separator, and assuming that the separator is an infinitely large liquid reservoir, a mechanical-energy balance leads to

$$\overline{P}_{dc, z=L} = \langle \overline{P} \rangle_{z=L} - 0.75 \rho_1 \overline{v}_{dc}^2$$
 (6)

On the other hand, a vertical momentum balance leads to (after using Eq. 6)

$$\langle \bar{P} \rangle_{z=L} = P_a + \rho_1 g h_t - \frac{\rho_1 \bar{\nu}_{dc}^2 D_{dc}^2}{D_{dc}^2 + D^2} \left[ 0.25 + \frac{D_{dc}^2}{D^2 (1 - \bar{\epsilon}_{z=L})} \right], \tag{7}$$

where  $P_a$  is the atmospheric pressure, and  $h_t$  is the liquid height in the separator. We have neglected the gas holdup in the separator, which is expected to be much smaller than that of the riser.

In our previous work (Sáez et al., 1998), the last term in the righthand side of Eq. 7 was neglected. This term can be of the same order of magnitude as the hydrostatic contribution ( $\rho_1 gh_t$ ). For the conditions explored in this work, however, each of these terms is less than 5% of the atmospheric pressure, and therefore its impact on predicted gas holdups and liquid circulation rates is negligible.

### Model implementation

The basic information that must be available to solve the system of equations presented earlier consists of the riser and downcomer lengths and diameters, physical properties of gas and liquid phases, the atmospheric pressure, the height of liquid in the gas/liquid separator, the bubble radius at the sparger, and the gas superficial mass velocity. Moreover, suitable equations to predict the two-phase wall friction factor in the riser ( $f_r$ ) and the gas/liquid drag coefficient ( $\hat{C}_d$ ) must be available.

The system of equations is a set of nonlinear algebraic equations plus an ordinary differential equation for the pressure (Eq. 3) whose boundary condition is Eq. 7. The differential equation was discretized by means of a Runge-Kutta technique, and the system of equations is solved by means of iterative procedures described in detail by Sáez et al. (1998).

Sáez et al. (1998) analyzed three different gas/liquid drag-coefficient correlations and obtained similar results with all of them. The equation presented by Ishii and Zuber (1979), valid for  $Re < 2 \times 10^5$ , was used in this work:

$$\hat{C}_d = \frac{24}{R_0} (1 + 0.1 Re^{3/4}). \tag{8}$$

In this equation, *Re* is the local Reynolds number, based on the relative velocity between gas and liquid phases.

The local two-phase wall friction factor is predicted by the correlation developed by Akita et al. (1988):

$$f_r = \frac{0.187\sqrt{\bar{\epsilon}}}{(1-\bar{\epsilon})^{0.1}} \left( \frac{\sqrt{gD}}{\langle \bar{\nu}_1 \rangle} \right)^{1.1}.$$
 (9)

This correlation is empirical, and does not reduce to the limit of single-phase flow as the gas holdup becomes very small. Under the conditions explored in this work, the friction factors predicted by this equation are appreciably higher than those for single-phase flow.

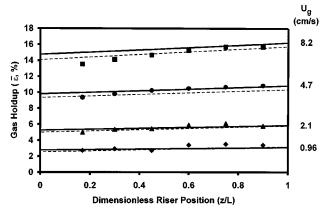


Figure 2. Gas holdup profiles.

Downcomer diameter: 8.9 cm. Data points, Young et al. (1991); solid lines, this work; dashed lines, Sáez et al. (1998).

#### **Results and Discussion**

Figure 2 shows a comparison between the model developed in this work and the experimental results of Young et al. (1991) for the 8.9-cm downcomer. The present model adequately represents the experimental data, including a slight increase of gas holdup with height, which is a consequence of the expansion of the gas as the pressure decreases. The fit between the model and experimental data is close to that obtained by Sáez et al. (1998) (dashed lines), in which the friction factors in the downcomer given by Young et al. (1991) were used, and losses associated with the gas/liquid separator were ignored.

The model results in Figure 2 employ Eq. 9 in the calculation of the friction factor in the riser. Young et al. (1991) measured pressure drops between two points in the riser, and found that the predicted values of  $f_r$  adequately matched these pressure drops for the conditions obtained with the 8.9-cm downcomer. However, for the 14-cm downcomer, the measured pressure drops corresponded to a friction factor that was approximately twice that calculated from Eq. 9 (see Young et al., 1991, Table 4). This might reflect the inadequacy of this correlation at the higher liquid velocities obtained with the 14-cm downcomer (see below). For this reason, in modeling the 14-cm downcomer, we predicted  $f_r$  as twice the value given by Eq. 9.

Figure 3 shows gas holdup profiles for the 14-cm down-comer. The model provides a better representation of the experimental data than that obtained by Sáez et al. (1998), although the data are still underpredicted to some extent.

The liquid-circulation fluxes for the experiments in the two downcomers are presented in Figure 4, along with the present model predictions and the predictions of the model of Sáez et al. (1998). The higher liquid fluxes for the 14-cm downcomer seen in Figure 4 are a consequence of the higher liquid flow rate caused by the increase in downcomer cross-sectional area.

The results obtained demonstrate how a careful consideration of drag and frictional effects in the system can lead to an accurate representation of experimental data. A particular improvement over the previous version of the model is evident in the predictions for the 14-cm downcomer. As shown in Figures 3 and 4, the predictions of Sáez et al. (1998) gener-

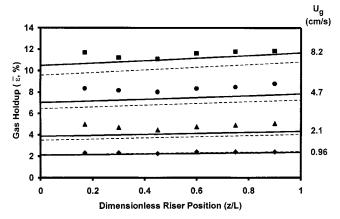


Figure 3. Gas holdup profiles.

Downcomer diameter: 14 cm. Data points, Young et al. (1991); solid lines, this work; dashed lines, Sáez et al. (1998).

ally underestimated gas holdups and overestimated liquid circulation rates. A readjustment of energy losses in the downcomer and gas/liquid separator was not enough to improve the overall representation of the experimental data for both downcomer diameters. The use of an adequate value of the friction factor in the riser, in conjunction with the more detailed model of the downcomer and gas/liquid separator has led to a considerable improvement in the model predictions.

Table 1 shows a comparison of the relative magnitude of the contributions of the different accessories and frictional elements in the system to mechanical energy losses, for one gas superficial velocity. All elements have a significant contribution. For the 8.9-cm downcomer, losses associated with the downcomer itself dominate, whereas for the 14-cm downcomer, losses associated with the riser have the greatest impact. Notice that riser friction goes from being 7% of total energy losses in the 8.9-cm downcomer to 25% in the 14-cm downcomer. The reason for this is mainly the higher liquid velocities that are obtained in the riser with the larger downcomer (as observed in Figure 4). The importance of losses in the accessories (tee, elbow, contractions and expansions,

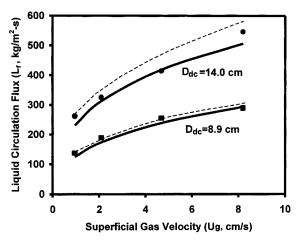


Figure 4. Liquid circulation flux and global gas holdups.

Data points, Young et al. (1991); solid lines, this work; dashed lines, Sáez et al. (1998).

Table 1. Relative Magnitude of Drag and Frictional Losses for  $U_g = 8.2 \text{ cm/s}$ 

	$D_{dc} = 8.9 \text{ cm}$	$D_{dc} = 14 \text{ cm}$
Downcomer wall friction	14	5
Contraction separator/downcomer	23	13
Elbow	16	9
Expansion downcomer/riser	27	6
Tee	5	15
Expansion riser/separator	3	10
Vanes	5	17
Riser wall friction	7	25

Note: The losses are expressed as a percentage of each element contribution with respect to total losses in terms of the coefficient multiplying  $\bar{v}_{do}^2/2 g$  in the mechanical energy balance.

vanes) will depend on the reactor scale. In the pilot-scale system explored in this work, they are extremely relevant. In an industrial reactor, the increased length of riser and downcomer would make the wall friction terms relatively more im-

To explore the sensitivity of the model to individual loss terms, we have compared the model predictions for the 14-cm downcomer when two of the terms are changed. First, the loss coefficient for the elbow has been increased from  $K_E$ = 0.35 to  $K_E = 0.7$ . The new value would correspond to two 45° elbows in series instead of one, long-radius elbow. Second, the loss coefficient for the "tee" has been changed from  $K_T$ = 1.5 to  $K_T$  = 1.8. These changes are within the ranges of loss coefficients given in the literature. The changes lead to an increase in mechanical energy losses, which results in a reduction in liquid circulation flux and a consequent increase in the global gas holdup. However, the magnitude of the changes is relatively small in the range of superficial gas velocities explored (0-8 cm/s): the predicted liquid circulation flux is reduced by less than 5%, while the global gas holdup increases less than 1%.

All the results generated with the model considered a bubble radius at the sparger of 1 mm. This value was found by Sáez et al. (1998) to yield the most accurate fit of the data of Young et al. (1991). It might be argued that this parameter can vary, given the range of conditions covered in this work. For instance, increasing the superficial gas velocity increases bubble size because of higher gas momentum during bubble growth (Geary and Rice, 1991). In the gas-lift configuration, however, an increase in gas velocity leads to a higher gas holdup in the riser and, consequently, a higher liquid circulation velocity. It is expected that higher liquid velocities lead to earlier bubble detachment due to increases in the gas/liquid drag force (Márquez et al., 1999a). This can somewhat compensate for the effect of gas momentum. In other words, even though the range of velocities analyzed in this work is relatively wide, variations in sparger bubble size might not be significant.

#### Concluding Remarks

In this work we show how predictions of the hydrodynamics of gas-lift reactors can be improved by careful quantifica-

tion of mechanical-energy losses in the system. The results suggest that drag and friction in all elements of the system (including straightening vanes and gas/liquid separator) must be taken into account in the formulation. On the other hand, downcomer friction factors can be predicted from correlations for fully developed, single-phase turbulent flow, and riser-wall friction must be adequately quantified. The results also show that mechanical-energy losses in the various accessories of the system are the key elements that determine the liquid-circulation velocity for the reactor scale analyzed in this work.

#### **Notation**

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L = riser length, m
L_{dc} = downcomer length, m
  r_b = bubble radius, m
  R = \text{gas constant}, \text{ m}^3 \cdot \text{Pa/K} \cdot \text{kmol}
 Re = \text{Reynolds number}, \ Re = \rho_1(\langle \bar{v}_g \rangle - \langle \bar{v}_1 \rangle) 2 r_b / \mu_1
  T= temperature, K
 U_{\sigma} = inlet gas superficial velocity evaluated at atmospheric pressure,
           = GRT/(P_aM_g), m/s
  V_i = volume of phase (i = g, I), m<sup>3</sup>
  z= axial coordinate, m
\mu_1 = liquid viscosity, Pa·s
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## Other symbols

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\langle x \rangle = denotes volume average of x
\langle \bar{x} \rangle = denotes cross-sectional average of \langle x \rangle
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